

# MATH7421 Coursework 4

This coursework is worth 2% of your mark for this module and should be handed in by **12pm** on **Friday 27th November**. Remember to show your working!

1. **(5 marks)** Show that  $\alpha = \sqrt{2} + \sqrt{5}$  is an algebraic number and find the minimal polynomial  $m_\alpha(x)$  with proof. Hence write down the degree of the number field  $\mathbb{Q}(\sqrt{2} + \sqrt{5})$  and give a full description of its elements.
2. **(5 marks)** Let  $d \neq 0, \pm 1$  be a square-free integer. Show that the ring of integers of the number field  $K = \mathbb{Q}(\sqrt{d})$  is given by:

$$\mathcal{O}_K = \begin{cases} \mathbb{Z} \left[ \frac{1+\sqrt{d}}{2} \right] & \text{if } d \equiv 1 \pmod{4} \\ \mathbb{Z}[\sqrt{d}] & \text{if } d \not\equiv 1 \pmod{4} \end{cases}$$