

AMIS50 - Week 1

Welcome to the course!

A few things:

- ① I'm Dan and I'm new to USW.
- ② I'm going to be teaching you Maths for Mech/Aero engineers.
- ③ I want you all to not only do your best, but to enjoy doing it!
- ④ I'm friendly and so please ask me any questions you have, don't be shy.

Course Setup

- ① 3 hours/week - 2 lectures, 1 problems class
(Will try and give you enough breaks!)
- ② 25% Class Test in Dec, 25% MATLAB CWK in Term 2, 50% Exam (2hr)
- ③ Notes and exercises are on Blackboard.

What is this course about?

You all want to be Engineers, but this requires a firm grasp of many Mathematical tools.

We're going to learn those tools!

Algebra I

Recall - Four basic operations: $+, -, \times, \div$

$+, -$ are "inverses" $\times, \div$ are "inverses".

Nice rules: If a, b, c are numbers:

Commutativity: $a+b = b+a$
 $a \times b = b \times a$

Associativity: $a+(b+c) = (a+b)+c$
 $a \times (b \times c) = (a \times b) \times c$

Distributivity: $(a+b) \times c = a \times c + b \times c$

Indices If n is a natural number and a is a number then:

$$a^n = \underbrace{a \times a \times a \dots \times a}_{n \text{ times}}$$

Rules: $a^n \times a^m = a^{n+m}$ $a^n \div a^m = a^{n-m}$
 $(a^n)^m = a^{n \times m}$

[Notice I didn't write $n \times m$. This will happen from now on,]

Expressions

In algebra, we represent quantities by symbols.
 Combinations of symbols are expressions.

We can find values of expressions by substitution.

[Remember: BODMAS]

Ex 1.1 If $a=3, b=7$ then:

$$ab - a - b = 3 \times 7 - 3 - 7 \\ = 21 - 3 - 7 = 11$$

Ex 1.2 If $a=4, b=5, c=-4, x=2, y=3$

then: $x^2 + 3x + 7 = 2^2 + 3 \times 2 + 7 = 4 + 6 + 7 = 17$

$$\begin{aligned} \cdot \frac{(a^4 - 19a)^2}{bx^3 + b^2x} &= \frac{(4^4 - 19 \times 4)^2}{5 \times 2^3 + 5^2 \times 2} = \frac{(256 - 76)^2}{5 \times 8 + 25 \times 2} \\ &= \frac{180^2}{40 + 50} = \frac{180^2}{90} \\ &= 2 \times 180 = 360 \end{aligned}$$

Expressions can be simplified by grouping like terms.

$$\begin{aligned}\text{Ex 1.3} \quad a + 3a - b + 2b - 7 \\ = 4a + b - 7\end{aligned}$$

$$\begin{aligned}\text{Ex 1.4} \quad 3x^2 - 2x + 1 + x^2 + 4x + 5 \\ = 4x^2 + 2x + 6\end{aligned}$$

We can expand brackets using distributive law

$$\begin{aligned}\text{Ex 1.5} \quad (2+4)(3+5) &= 2 \times 3 + 2 \times 5 + 4 \times 3 + 4 \times 5 \\ (6 \times 8) &= 6 + 10 + 12 + 20 \\ &= 48 \quad \checkmark\end{aligned}$$

$$\begin{aligned}\text{Ex 1.6.} \quad & \bullet 2a(a+3b) = 2a^2 + 6ab \\ & \bullet (a+b)(a-b) = a^2 - ab + ba - b^2 \\ & \quad \quad \quad = a^2 - b^2\end{aligned}$$

We can go the other way... factorization.

$$\text{Ex 1.7} \quad 5x + 20 = 5(x + 4)$$

Ex. 1.8 . $7x + 14 = 7(x+2)$
 . $x^2 + 3x = x(x+3)$

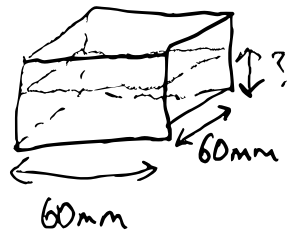
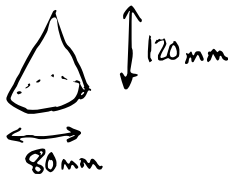
We can use indices rules to simplify fractions.

Ex 1.9. $\frac{x^2 y}{y^2 x} = x^{2-1} y^{1-2} = x y^{-1} = \frac{x}{y}$

Ex 1.10 $x^2 y z^3 \div \frac{3}{x y^2 z^2} = \frac{1}{3} x^{2-(-1)} y^{1-(-2)} z^{3-(-2)}$
 $= \frac{1}{3} x^3 y^3 z^5$

Equations - Describe relations between expressions.

Ex 2.1



Let the height be h , then since volume is constant:

$$\frac{1}{3} \pi \times 40^2 \times 120 = 60^2 \times h$$

$$64000 \pi = 3600 h \quad (\text{Eq}^{\wedge})$$

The unique solution is $h = \frac{64000}{3600} \pi = \frac{160}{9} \pi \approx 55.851 \text{ mm}$

We can rearrange equations to get whichever quantity we want.

Ex 2.2 If $V = \frac{4}{3}\pi r^3$ then $r^3 = \frac{3V}{4\pi}$

$$\text{so } r = \sqrt[3]{\frac{3V}{4\pi}}$$

Linear equations are the simplest, each quantity appears with only numbers as coefficients. Solving is quite easy.

Ex 2.3 If $6b - 5 = 10 + b$ then $5b = 15$

$$\text{so } b = \frac{15}{5} = 3$$

Ex 2.4 • If $\frac{4}{y} + 3 = 5$ then $\frac{4}{y} = 2$

$$\text{so } y = \frac{4}{2} = 2$$

• If $2x + 1 = 2x + 4$ then $1 = 4$ so no solutions.

• If $3x + x + 8 = 4x + 12 - 4$ then
 $4x + 8 = 4x + 8$

This is true for every value of x !

Algebra 2

Graphs of Equations

We can try to picture equations using graphs.

Why? It gives a rough idea of how fast/slow one quantity grows with respect to another.

Today - Our equations will be $y = f(x)$ with $f(x)$ an expression in x . Lots of equations can be rearranged to be of this form.

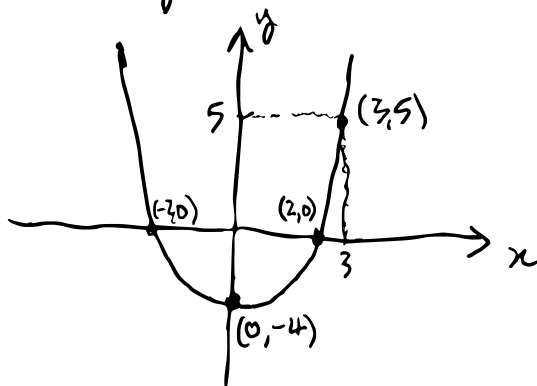
First: Need a table of pairs (x_0, y_0) that satisfy the equation:

Ex 4.1 $y = x^2 - 4$

x_0	-5	-4	-3	-2	-1	0	1	2	3	4	5
y_0	21	12	5	0	-3	-4	-3	0	5	12	21

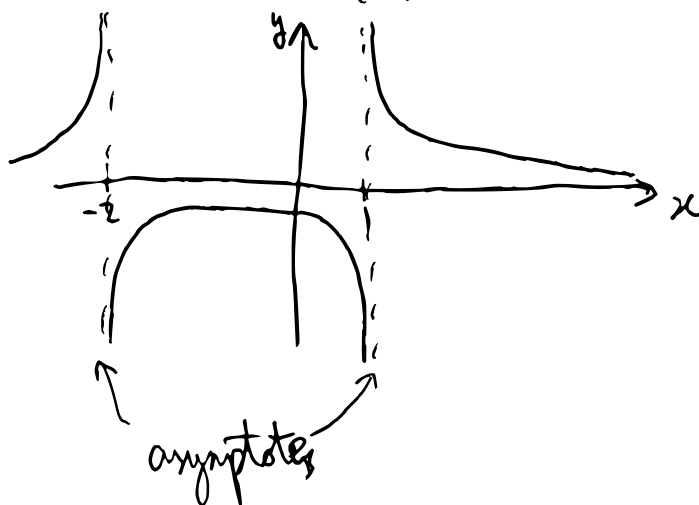
Second: We graph using Cartesian coordinates.

Ex 4.2 $y = x^2 - 4$



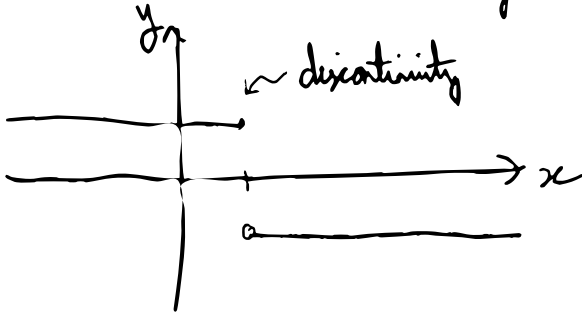
A linear equation $y = mx + c$ gives a line,
(if m, c fixed)
Tougher equations need more care!

Ex 4.3 $y = \frac{1}{(x-1)(x+2)}$

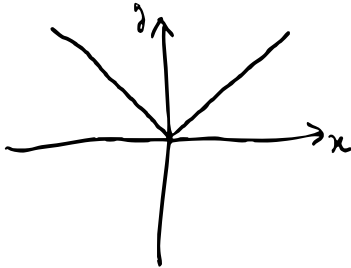


Sometimes, equations are given in pieces.

Ex 4.4 $y = \begin{cases} 1 & \text{for } x \geq 1 \\ -1 & \text{for } 1 > x \end{cases}$



Ex 4.5 $y = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$



Systems of Simultaneous Linear Equations

A single linear equation $y = mx + c$ has infinitely many solutions, but multiple such eq^s might have a unique solution.

First method: Substitution

Ex 5.1 If $3x + 2y = 8$ then
 $2x - 4y = -12$

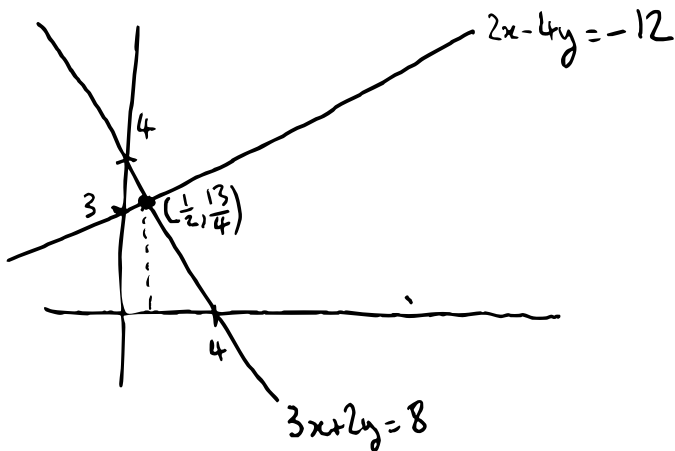
$Eq^1 \rightarrow y = \frac{8-3x}{2}$

$Eq^2 \rightarrow 2x - 4\left(\frac{8-3x}{2}\right) = -12$

$\rightarrow 2x - 16 + 6x = -12$

$\rightarrow 8x = 4$

$\rightarrow x = \frac{1}{2} \rightarrow y = \frac{13}{4}$



$$y = \frac{2x+12}{4}$$

$$= \frac{x}{2} + 3$$

Second method: Equate coefficients.

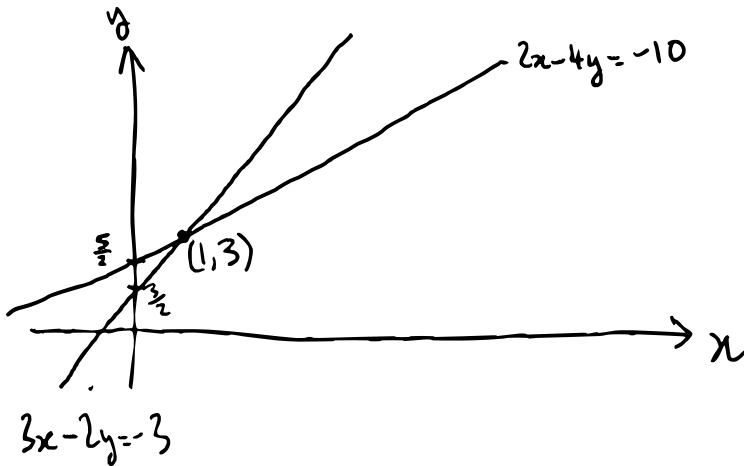
Ex 5.2 If $2x - 4y = -10$ then
 $3x - 2y = -3$

Eqⁿ 1 $\rightarrow 6x - 12y = -30 \quad (x3)$

Eqⁿ 2 $\rightarrow 6x - 4y = -6 \quad (x2)$

Subtract $\rightarrow -8y = -24$

$\rightarrow y = 3 \rightarrow x = 1$



$$y = \frac{2x+10}{4}$$

$$y = \frac{3x+3}{2}$$