MATH7421 Coursework 5

This coursework is worth 2% of your mark for this module and should be handed in by **12pm** on **Friday 11th December**. Remember to show your working!

1. **(5 marks)** Show that the ring $\mathbb{Z}[\sqrt{-13}]$ is not a UFD by giving, with proof, two inequivalent factorisations of 14 into irreducibles. You may assume that the units of this ring are $\mathbb{Z}[\sqrt{-13}]^\times = \{\pm 1\}$.

2. **(5 marks)**
   (a) Find a unit $u \in \mathbb{Z}[\sqrt{6}]$ satisfying $u > 1$.
   (b) Either prove that your unit is the smallest such unit, or find the smallest one.
   (c) Hence give a full description of the unit group $\mathbb{Z}[\sqrt{6}]^\times$.
   (d) Use part (c) to find three positive integer solutions to the Pell equation $x^2 - 6y^2 = \pm 1$.